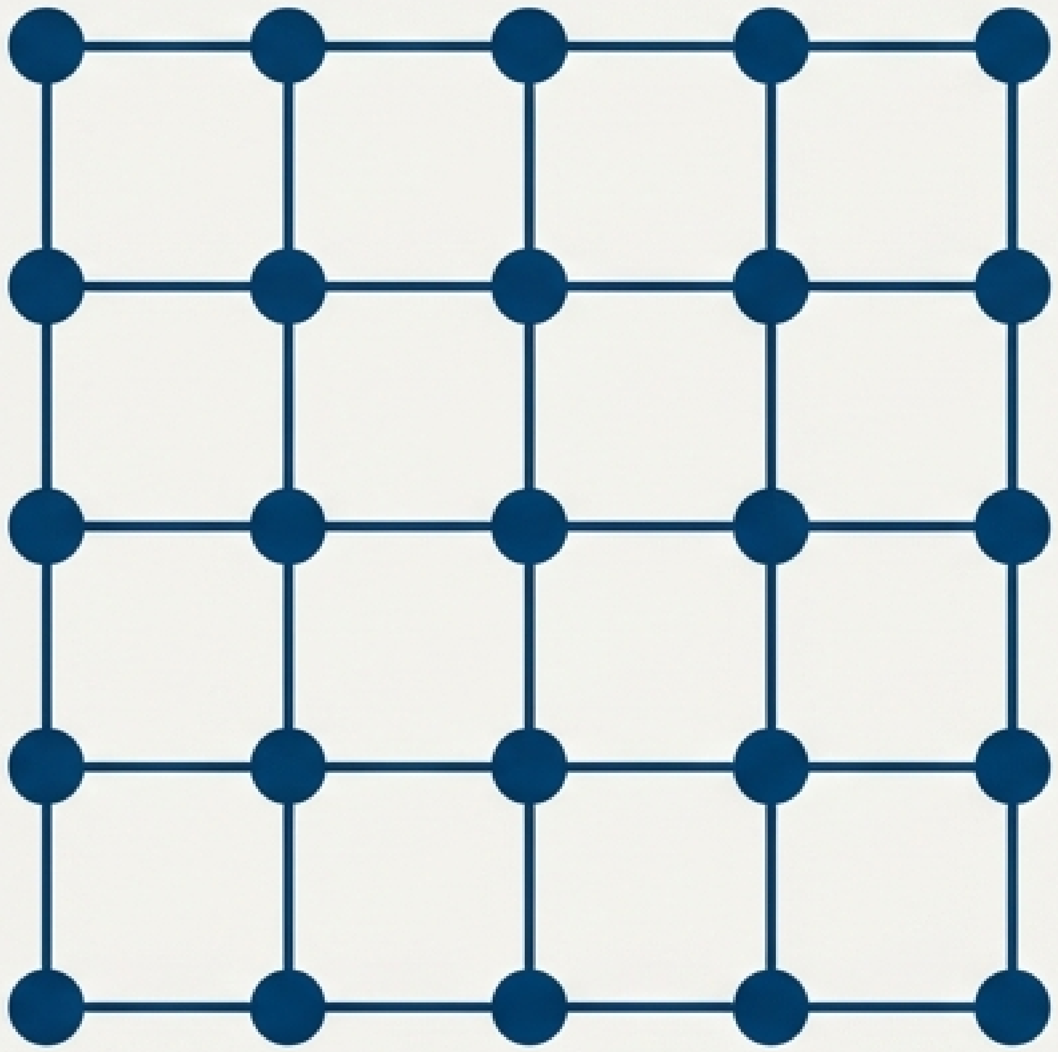
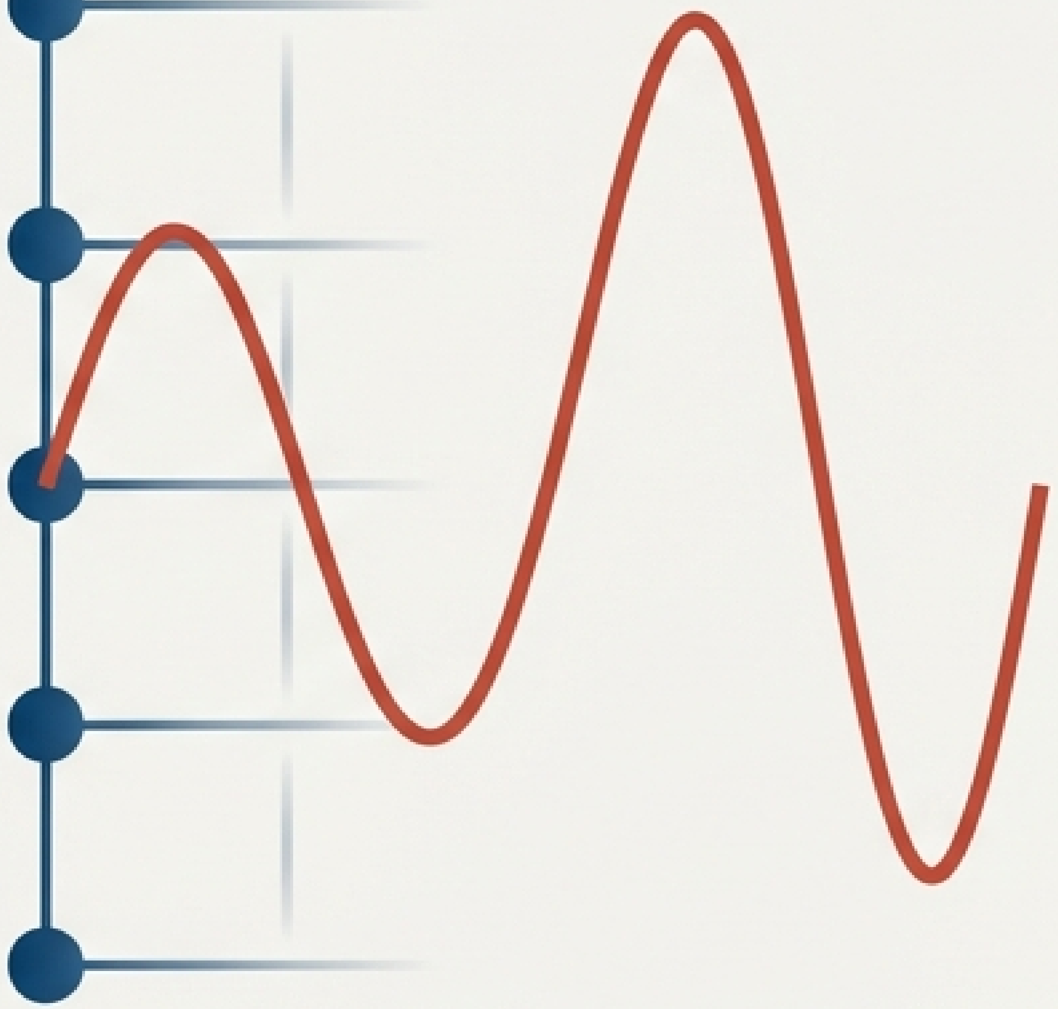
The Structural Origin of Schrödinger-Type Dynamics

A First-Principles Derivation from JS-SH Discrete Geometry



Discrete Structure (JS-SH)

Premise: We demonstrate that Schrödinger dynamics emerge inevitably as the structure-hiding continuum limit of a simpler system: a real, two-channel, norm-conserving discrete network.



Continuum Limit (Schrödinger)

Based on the work of Seunghyun Hong (Jan 2026)

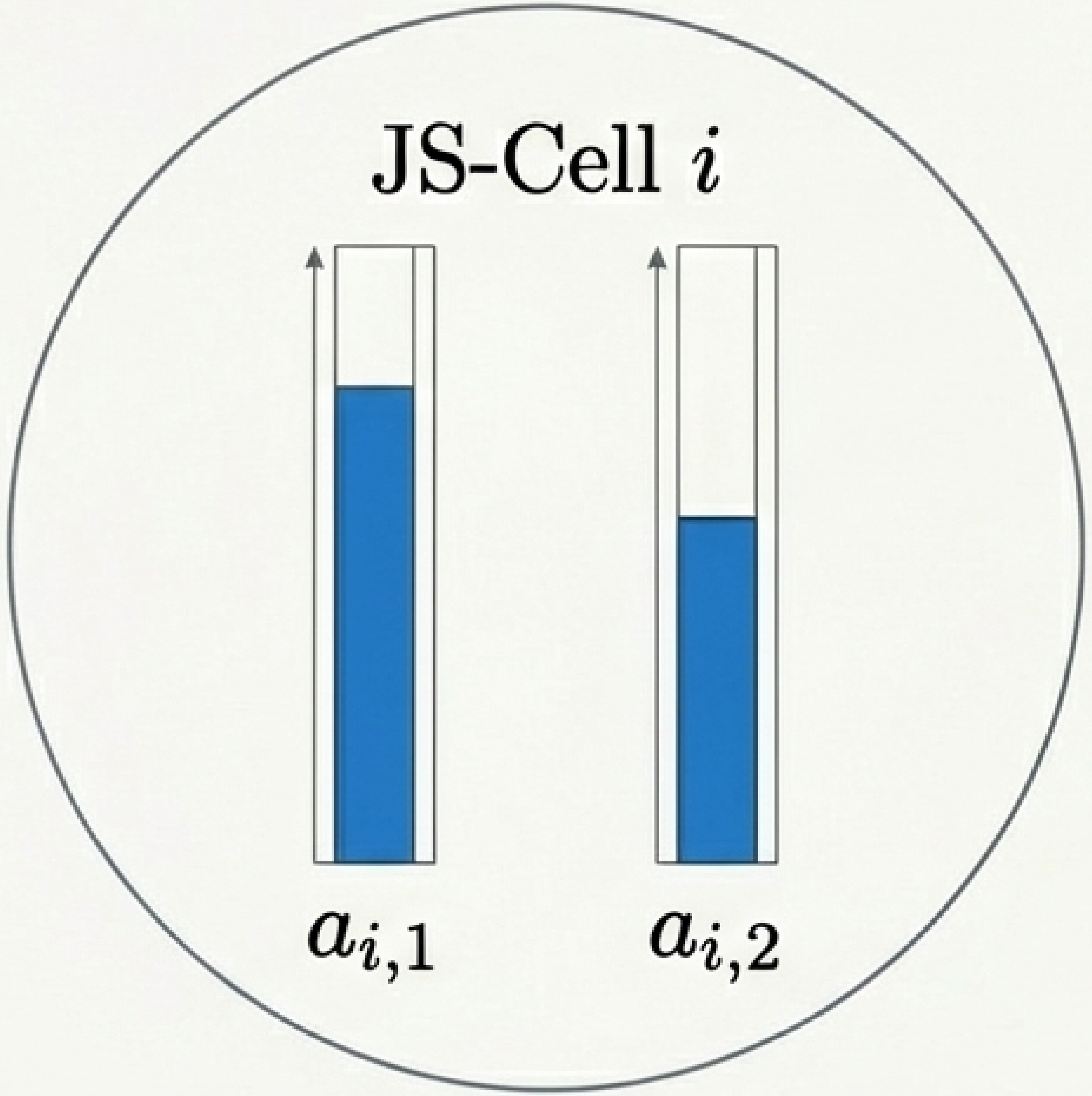
Axiom vs. Emergence

Standard Formulation	Structural Interpretation
Status: **Fundamental Postulate**	Status: **Effective, Structure-Hiding Representation**
Direction: **Equation → Interpretation**	Direction: **Structural Constraints → Schrödinger Equation**
Complex Numbers (i): **Axiomatic / Assumed**	Complex Numbers (i): **Emergent Generator of Rotations (J)**
Wavefunction: **Primitive Complex Scalar**	Wavefunction: **Compressed Real Vector**

Core Insight: We do not assume quantum mechanics. We identify the minimal structural ingredients required for Schrödinger dynamics to appear.

The Primitive Ingredient: The JS-Cell

A Real, Two-Channel State. No Complex Numbers.



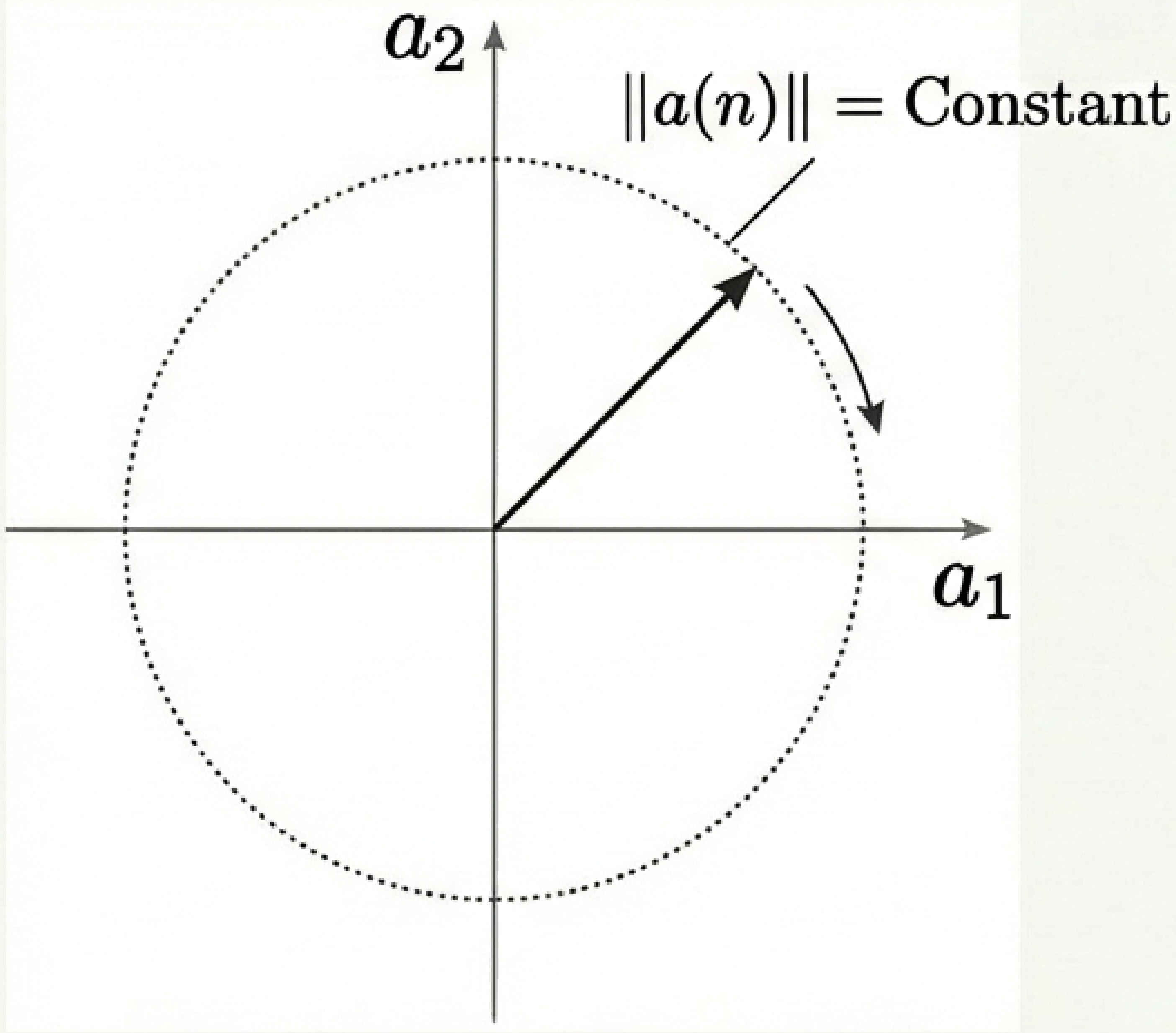
The State Vector:

$$\boldsymbol{a}_i(n) = \begin{pmatrix} a_{i,1}(n) \\ a_{i,2}(n) \end{pmatrix} \in \mathbb{R}^2$$

- Discrete time steps n
- Strictly Real-valued channels
- **Minimality**: Two channels are the structural minimum required to support continuous rotation.

The Governing Law: Strict Norm Conservation

Global structural norm must be invariant under time evolution.



The One Rule:

$$N(n) = \sum_i \|a_i(n)\|^2 = \text{const}$$

The Implication:

For any local linear update $a(n+1) = Ma(n)$, conservation requires:

$$M^T M = I$$

Result: Dynamics are restricted to the Orthogonal Group O(2).

The Structural Origin of Phase

Deriving the Generator Matrix **J**

Step 1:
Any rotation $R(\theta)$ for small steps Δt can be written as:

$$R(\theta) \approx I + \omega \Delta t J$$

Step 2:
This yields the unique generator matrix **J**:

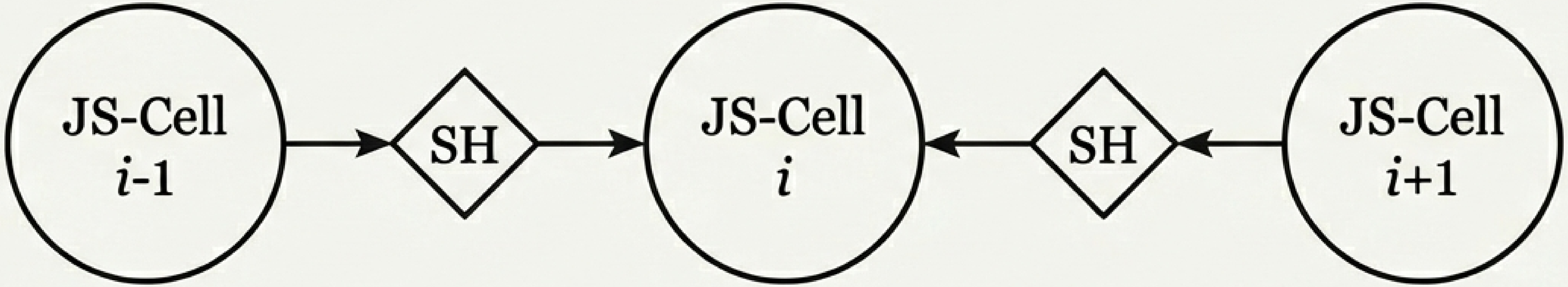
$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Step 3 (The Reveal):
Calculate the square of **J**:

$$J^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

Insight: **J** is a real rotation operator. It satisfies $x^2 = -1$ without being an imaginary number. This is the physical mechanism that complex notation eventually hides.

Connectivity: The SH-Hub and the Laplacian



Mechanism:

1. **Locality**: Interaction only with nearest neighbors.
2. **Linearity**: Simple mixing.
3. **Symmetry**: Left/Right balance.

The Emergent Operator:

These conditions uniquely select the Discrete Laplacian:

$$(\Delta_d a)_i = a_{i+1} - 2a_i + a_{i-1}$$

The Exact Discrete Update Law

Global Update Step

$$A(n+1) = \exp(\Delta t \cdot J_g \cdot G)\, A(n)$$

J_g (Internal)

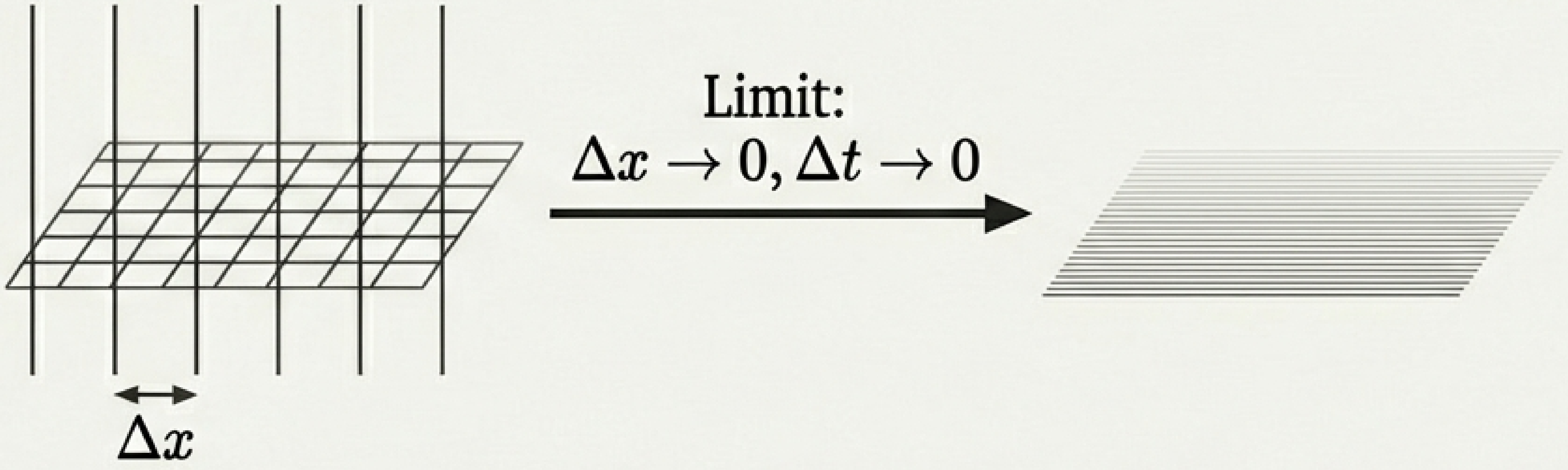
- Block-diagonal matrix containing J.
- Generates local rotation.
- Antisymmetric.

G (Spatial)

- Adjacency operator containing the Laplacian.
- Generates spatial mixing.
- Symmetric.

Since the product $J_g G$ is antisymmetric, the exponential is Orthogonal. This guarantees strict norm conservation.

The Ordered Continuum Limit



Expansion of the discrete update yields the first-order evolution:

$$\partial_t a(x,t) = J\left[\omega_0 a(x,t) + D\partial_x^2 a(x,t)\right]$$

Why ∂_t and not ∂t^2?

Locality + Norm Conservation + Memory-less updates forces first-order time evolution.

Higher order derivatives would imply non-local memory or extra internal degrees of freedom.

The "Complex" Repackaging

Translation Layer: From Real Geometry to Complex Notation

Translation Dictionary

	Real Vector Space	Notation Mapping	Complex Scalar Space
State:	$\boxed{a = (a_1, a_2)}$	$\Rightarrow$	$\boxed{\psi = a_1 + i a_2}$
Operator:	$\boxed{J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}$	$\Rightarrow$	$\boxed{i = \sqrt{-1}}$
Equation:	$\boxed{\partial_t a = J\,[\ldots]}$	$\Rightarrow$	$\boxed{\partial_t \psi = i\,[\ldots]}$

The imaginary unit 'i' is simply a shorthand for the real rotation matrix J. The physics remains entirely real.

Emergence of the Schrödinger Equation

Structural Scale: A conversion factor between frequency and energy.

$$i\hbar_{eff}\partial_t\psi = -\frac{\hbar_{eff}^2}{2m_{eff}}\partial_x^2\psi + V(x)\psi$$

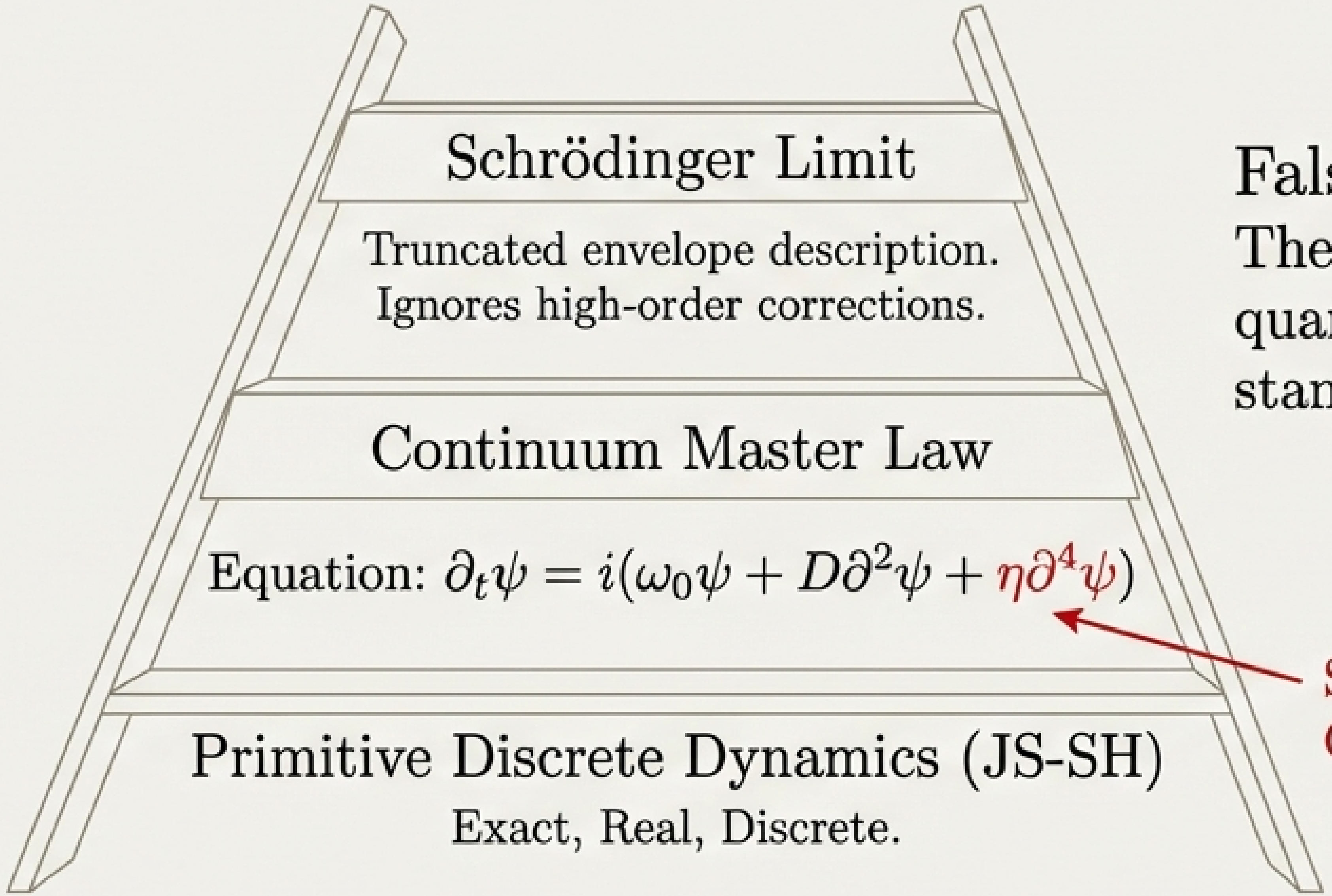
Effective Mass: Inverse of the diffusion/coupling coefficient D.

Potential: Local phase rotation rate $\omega_0(x)$.

Conclusion: We did not postulate this equation. We found it hiding inside the discrete structure.

The Ladder of Accuracy

Standard QM is an approximation.

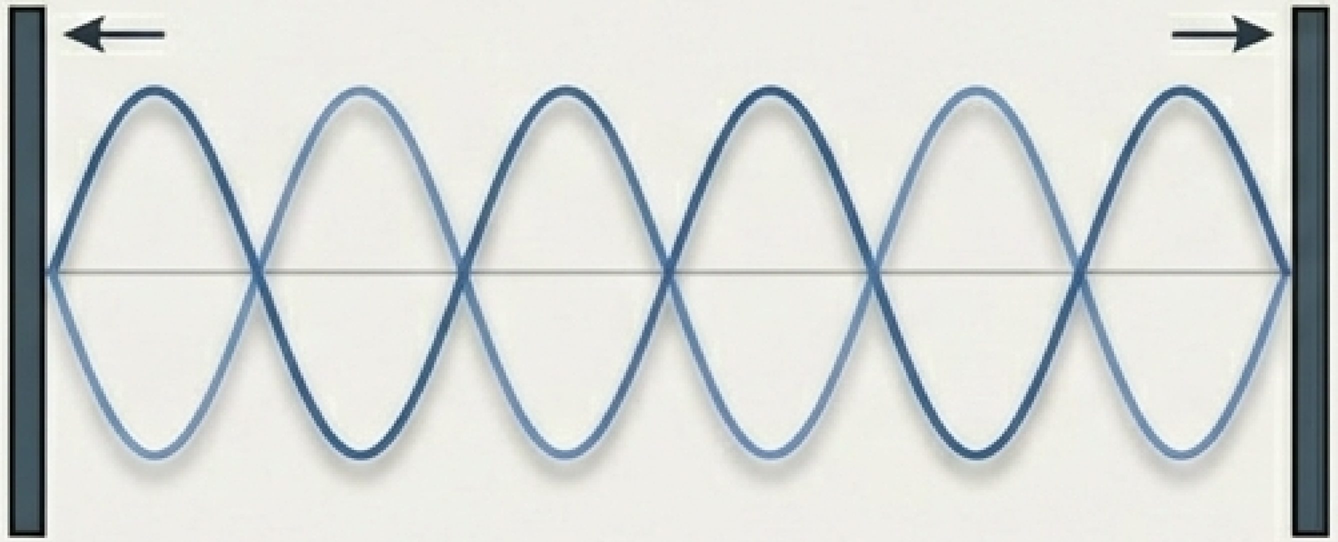


Falsifiability:
The JS-SH structure predicts quartic corrections (k^4) that standard QM does not.

Phase Closure vs. The Envelope

What do the solutions physically represent?

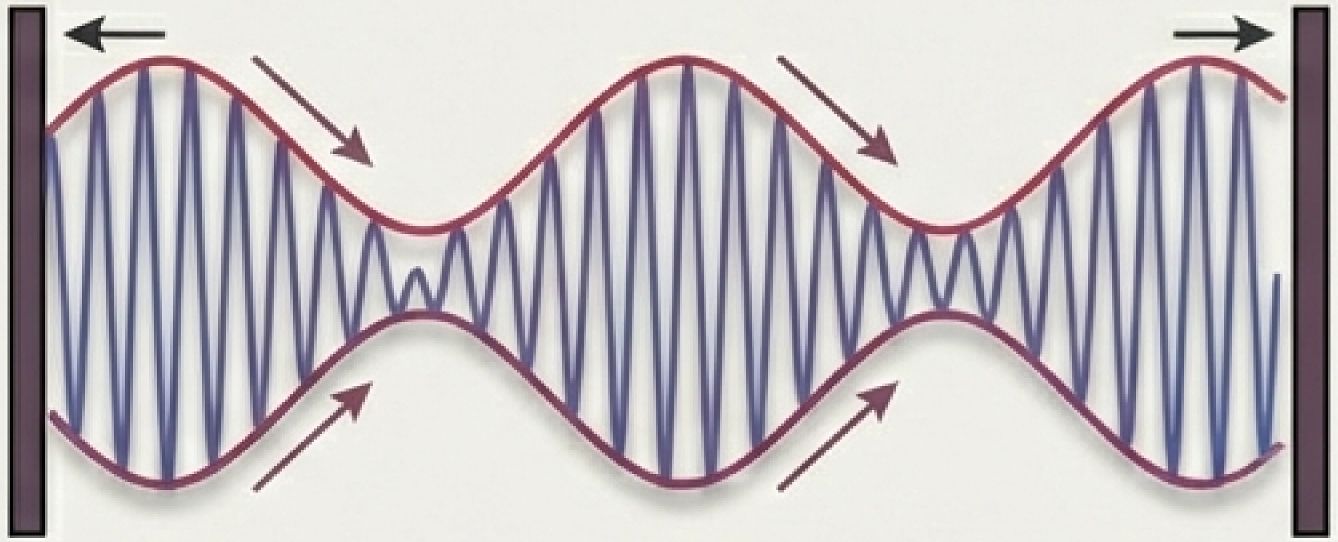
A. Phase-Closed



Integer Wavelengths = Eigenmodes

System locks into stationary states.

B. Phase-Open

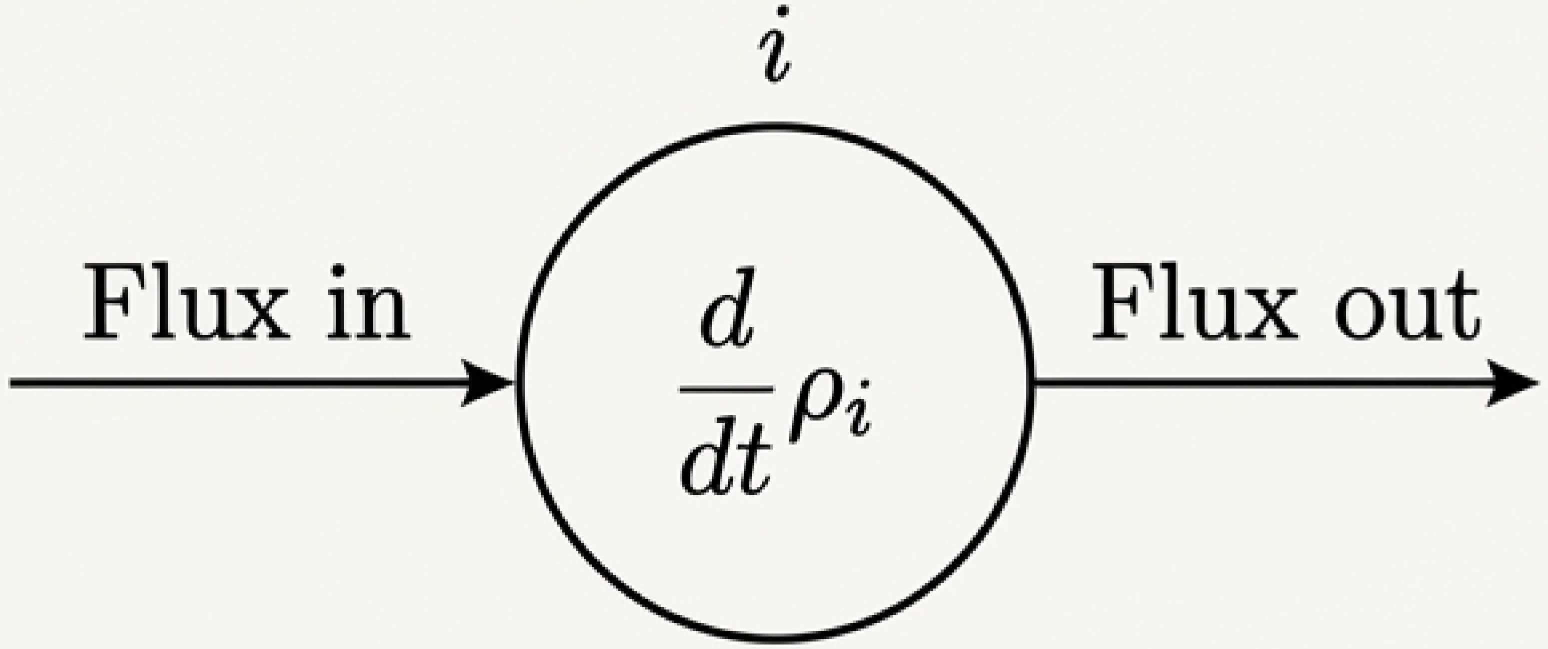


Mismatch = Schrödinger Evolution

Residual phase mismatch drives the envelope dynamics.

Insight: Schrödinger solutions are the envelope dynamics of structurally incomplete wave processes.

Unhiding the Structure: The Continuity Test



Site-wise Continuity Equation:

$$\frac{d}{dt}\rho_i = \sum_j \mathrm{Flux}_{ij}$$

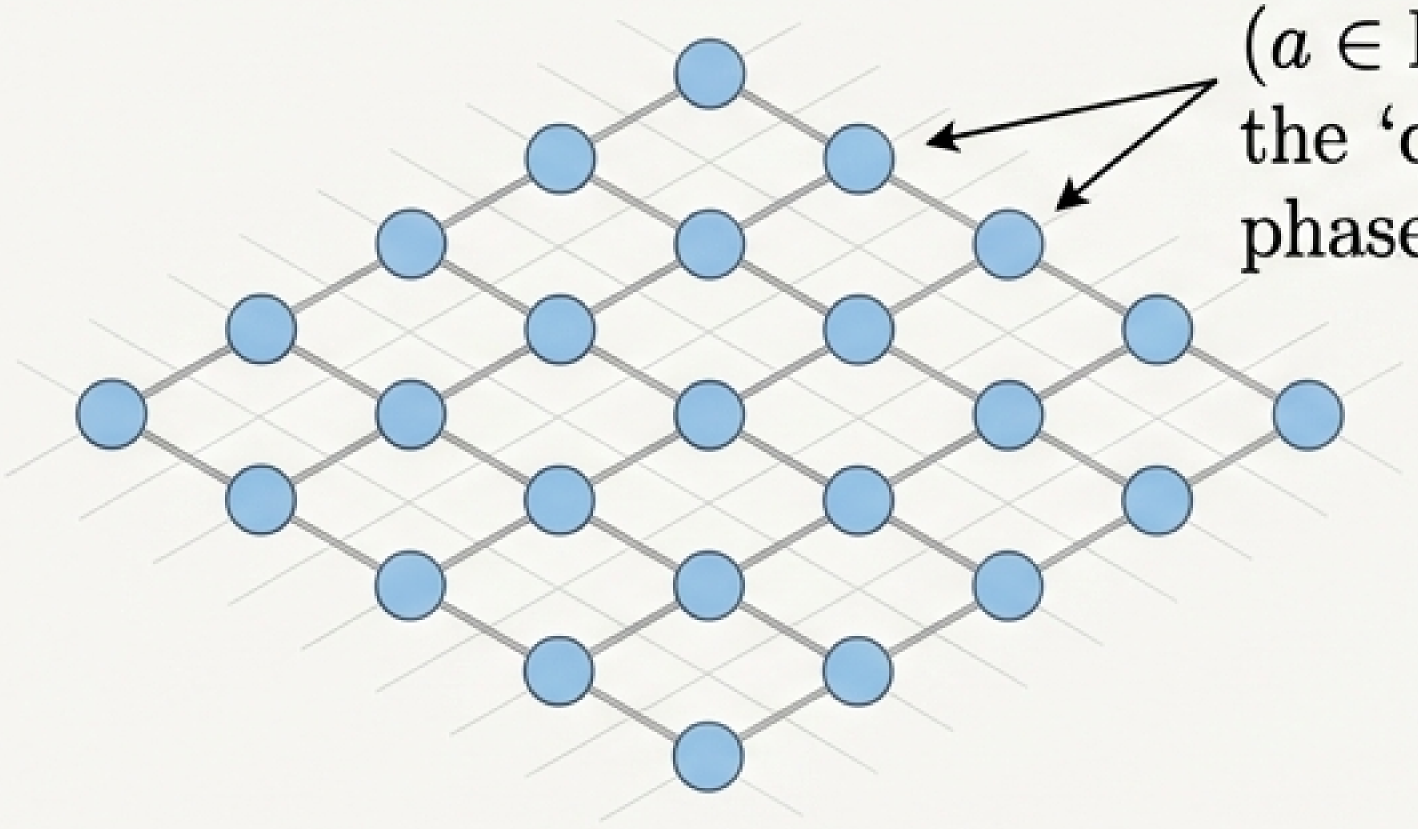
The JS-SH Signature Flux:

$$J_{ij} = 2\gamma \boldsymbol{a}_i^T \boldsymbol{J} \boldsymbol{a}_j$$

Interpretation: Directed Phase Area Exchange.

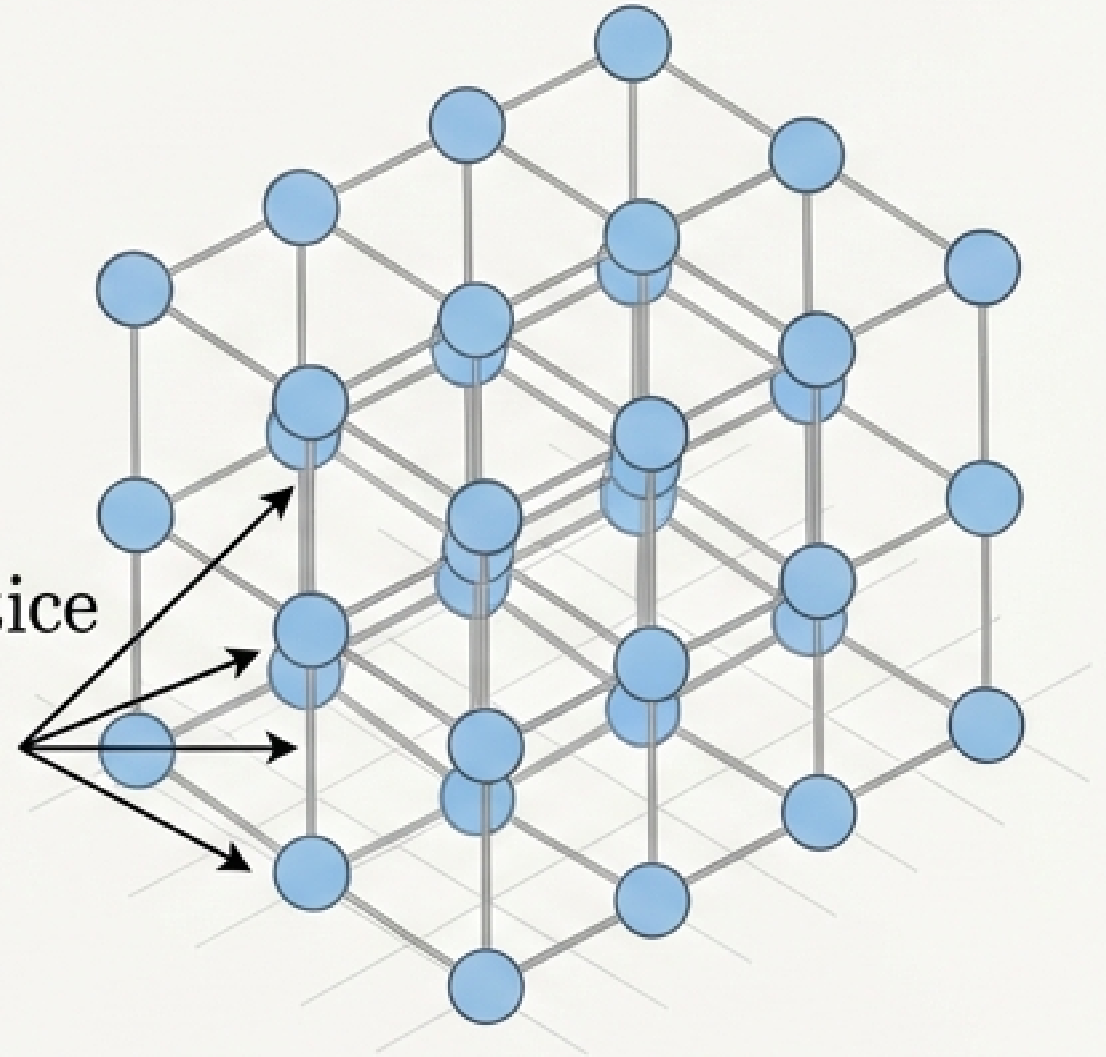
Global conservation is not enough. To prove a system is JS-SH, it must satisfy this specific local flux condition.

Extension to Higher Dimensions



The Internal State ($a \in \mathbb{R}^2$) handles the 'quantum' phase nature.

The External Lattice (d-dimensions) determines the geometry.

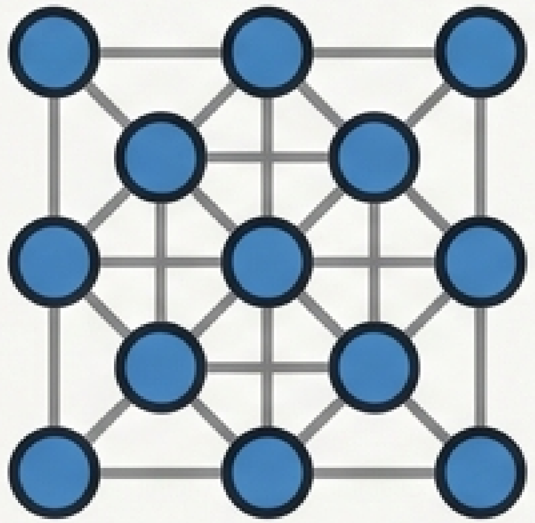


The derivation holds for $x \in \mathbb{R}^d$.

The derivation holds for $x \in \mathbb{R}^d$. The Laplacian ∇^2 simply adapts to the dimension.

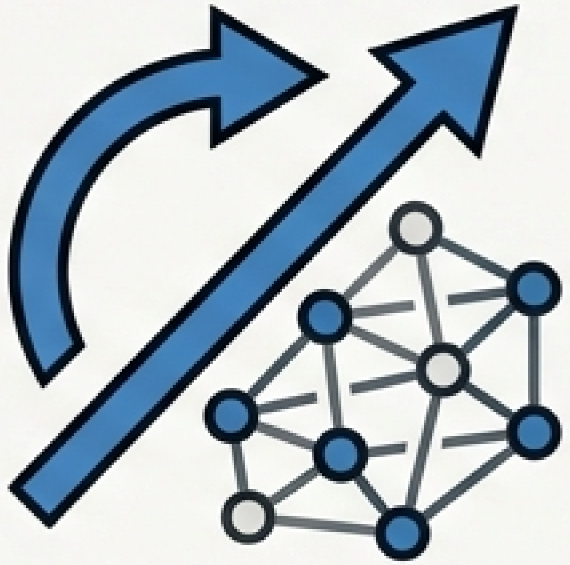
$\rightsquigarrow$ Insight: The framework is independent of the spatial dimension d.

Summary: The Structure Behind the Symbol



Primitive
JS-SH Cells

Constraint:
Norm
Conservation



Mechanism:
Rotation (J) + Mixing (L)

Continuum
Limit

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi}$$

Result:
Emergent Dynamics

The Schrödinger equation is a structure-hiding map of a simpler, real, discrete, and deterministic universe.